THE APPLICATION OF STATE FEED BACK CONTROL SYSTEM FOR STABILIZE INVERTER PENDULUM

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ABSTRACT

*Actually, inverter pendulum position is unstable. It is possible to find equilibrium condition but there is unstable. Under position control system it is feasible to find a stable condition. So we can overcome the unstable problem. Even though the system was getting any disturbances but under stability control the system look for its equilibrium. The control position also consists of DC motor and other devices. Analyzing the system we applied **state feed back** for **stabilize inverter pendulum.** The concept of this system can be developing for unstable object like a rocket position. Many unstable The Control System of this "**Inverter Pendulum"** is realized under digital signal processing controlling DC motor.*

INVERTER PENDULUM SYSTEM

Inverter pendulum absolutely unstable. It derived by movable cart at the rail. By using a belt the pendulum have fix connection to DC motor and encoder-1 as in figure 1 below.

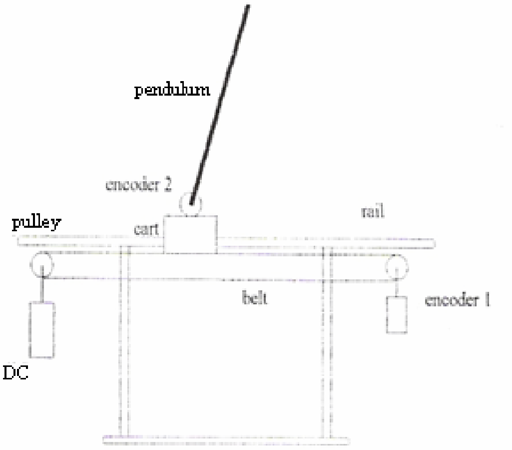

Fig.1. Pendulum Model

There are 2 encoder, it called encoder-l and encoder-2. Encoder-l is used for measure horizontal position of cart, and encoder-2 is used for measure angle position. In this control system the problem was :

How to control inverter pendulum position in stability?

In the method of this research it need a signal processing. The function signal processing are:

- instrumentation measurement,
- signal processing, and
- control voltage input parameter of DC motor.

As a instrumentation measurement, it read both cart horizontal position and angle pendulum position. Calculating a value of voltage parameter input. More friendly as a signal processing, we used personal computer.

Physical and Mathematical Model

Let we consider a physical and mathematical model of the pendulum control system as a figure 2 below.

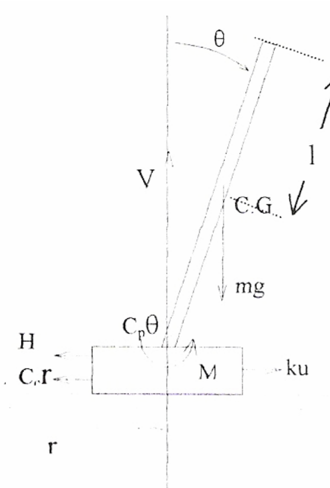

Fig.2. Physical Model of Pendulum

From Newton's Law, we derived the dynamic equations :

$$M\ddot{r} = ku - c_c\dot{r} - H \qquad (1)$$

M = mass of cart [kg]
r = horizontal displacement of cart [m]
u(t) =control voltage of motor [volt]

k = torque gain [N/volt]
c_c = bisques friction coefficient [kg/sec]
H = force at pendulum base in horizontal direction [N]
J = moment inertia of pendulum [kgm^2]
θ = angle of pendulum [rad]
V = force of to pendulum in vertical direction [N]
H = force of to pendulum in horizontal direction [N]
L = distance between joint and C.G [m]
c_p = bisques frictional coefficient of joint [kg^2/sec]
m = mass of pendulum [kg]
g = gravitation acceleration [m/sec^2]

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m = mass of pendulum [kg]
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Moment equation of pendulum at joint to CG:

$$J\ddot{\theta} = Vl\sin\theta - Hl\cos\theta - C_p\dot{\theta} \qquad (2)$$

Horizontal CG pendulum acceleration equation:

$$H = m\frac{d^2}{dt^2}(t + l\sin\theta) \qquad (3)$$

Vertical CG pendulum acceleration equation:

$$m\frac{d^2}{dt^2}(l\cos\theta) = V - mg \qquad (4)$$

from equation 3 and 4:

$$mr + ml\cos\theta - ml\sin\theta\theta^2 = H \qquad (5)$$

$$-ml\cos\theta\theta^2 - ml\sin\theta\theta = V - mg \qquad (6)$$

from equation 1, 2, 5 and 6:

$$(M+m)\ddot{r} + ml\cos\theta\ddot{\theta} + c_c r - ml\sin\theta\dot{\theta}^2 = ku \qquad (7)$$

$$(ml\cos\theta)\ddot{r} + (J + ml^2)\ddot{\theta} + c_p\dot{\theta} - mgl\sin\theta = 0 \qquad (8)$$

Equation (7) and (8) can be changed to matrix mode like:

$$\begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & J+ml^2 \end{bmatrix}\begin{bmatrix} \ddot{r} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -c_c\dot{r} + ml\sin\theta\dot{\theta}^2 + ku \\ -c_p\dot{\theta} + mgl\sin\theta \end{bmatrix} \qquad (9)$$

$$\begin{bmatrix} \ddot{r} \\ \ddot{\theta} \end{bmatrix} = \frac{1}{(M+m)(J+ml^2) - m^2l^2\cos^2\theta}\begin{bmatrix} J+ml^2 & -ml\cos\theta \\ -ml\cos\theta & M+m \end{bmatrix}\begin{bmatrix} -c_c\dot{r} + ml\sin\theta\dot{\theta}^2 + ku \\ -c_p\dot{\theta} + mgl\sin\theta \end{bmatrix}$$

Because the equilibrium state is at r = 0 and θ = 0. Also θ quite small and so assume that sin θ = θ, cos θ = θ and $\dot{\theta}^2 = 0$. So equation (9) can be linierized as:

$$\begin{bmatrix} \ddot{r} \\ \ddot{\theta} \end{bmatrix} = \frac{1}{(M+m)J - Mml^2}\begin{bmatrix} J+ml^2 & -ml \\ -ml & M+m \end{bmatrix}\begin{bmatrix} -c_c\dot{r} + ku \\ -c_p\dot{\theta} + mgl\sin\theta \end{bmatrix} \qquad (10)$$

$$= \frac{1}{\alpha_0}\begin{bmatrix} -m^2l^2g & -c_c(J+ml^2) & mlc_p & (J+ml^2)k \\ (M+m)mgl & mlc_c & -c_p(M+m) & -mlk \end{bmatrix}\begin{vmatrix} \dot{r} \\ \dot{\theta} \end{vmatrix} \qquad (11)$$

where $\alpha_0 = (M+m)J - Mml^2$

State Vector and State Equation

There are four states variable in the pendulum control system, it consist of:

- r as displacement of cart
- θ as angle of pendulum
- $\dot{r}$ as velocity of cart of pendulum
- $\dot{\theta}$ as angular velocity of pendulum

Let the four states:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} \qquad (12)$$

we call this x vector state as a *state vector*. Accordance equation (11) it have can be find vector equation like:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix}x + \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix}u \qquad (13)$$

and in matrix : $\boxed{\dot{x} = Ax + Bu}$ (14)

where :

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix}$$

The elements of matrix A and matrix B will be founded from equation (11) as like :

$$\begin{bmatrix} a_{32} & a_{33} & a_{34} & b_3 \\ a_{42} & a_{43} & a_{44} & b_4 \end{bmatrix} = \frac{1}{\alpha_0}\begin{bmatrix} -m^2l^2g & -c_c(J+ml^2) & mlc_p & (J+ml^2)k \\ (M+m)mgl & mlc_c & -c_p(M+m) & -mlk \end{bmatrix}$$

The output vector is :

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x \quad (15)$$

and in matrix $\boxed{\mathbf{y} = \mathbf{C}\,\mathbf{x}}$
(16)

where : $\mathbf{C} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$

Open Loop Pendulum

Accordance with equation (14) and equation (16), we make a open loop block of pendulum as:

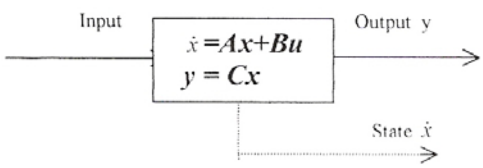

Fig.3 Block Diagram Pendulum

PARAMETERS AND STATE VALUES

As a result of measurement (except gravitation acceleration), it find that the values of parameter needs as below:

		Unity
M	0.7865	kg
k	0.7	N/volt
C_c	9.3	kg/sec
J	0.000163	Kgm^2
c_p	0.000157	kgm^2/s
m	0.03	kg
g	9.8	m/s^2
l	0.155	m

We have already know state equation:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix}$$

And by using $\alpha_0 = (M+m)J - Mml^2$

And then :

$$\begin{bmatrix} a_{32} & a_{33} & a_{34} & b_3 \\ a_{42} & a_{43} & a_{44} & b_4 \end{bmatrix} = \frac{1}{\alpha_0}\begin{bmatrix} -m^2l^2g & -c_c(J+ml^2) & mlc_p & (J+ml^2)k \\ (M+m)mgl & mlc_c & -c_p(M+m) & -mlk \end{bmatrix}$$

And by using all pendulum parameter constants, we can find that :

$$\begin{bmatrix} a_{32} & a_{33} & a_{34} & b_3 \\ a_{42} & a_{43} & a_{44} & b_4 \end{bmatrix} = \begin{bmatrix} -0.3027 & -11.71 & 0.00378 & 0.884 \\ 53.14 & 61.71 & -0.814 & 4.714 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.3027 & -11.71 & 0.00378 \\ 0 & 53.14 & 61.71 & -0.814 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0.884 \\ -4.17 \end{bmatrix}$$

BLOK DIAGRAM WITH STATE FEED BACK CONTROL

By using a state feed back control, the block diagram system as below:

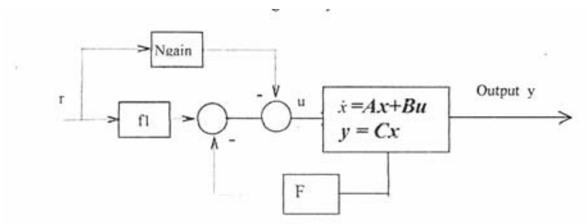

Fig.4. Block Diagram with State Feed Back Control

F is a state feed back, and

$$F = \begin{pmatrix} f_1 & f_2 & f_3 & f_4 \end{pmatrix}$$

In order to find a best response, we have to design feed back parameter. One of concepts is adjust the system's poles location. It should be done too many times until we find a suitable poles location which produce a best response. It can be done by using simulation program.

A suitable poles are:

$$p_1 = -400 + j20$$
$$p_2 = -400 - j20$$
$$p_3 = -50$$
$$p_4 = -2.9$$

Base on poles above ($p_1 = -400 + j20$; $p_2 = -400 - j20$; $p_3 = -50$; $p_4 = -2.9$), we can find a "feed back state". And it's feed back state value is:

$$F = (f_1 \quad f_2 \quad f_3 \quad f_4) = (-141 \quad -189 \quad -80.9 \quad -29.4)$$

Where

$$\mathbf{A}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.3027 & -11.71 & 0.00378 \\ 0 & 53.14 & 61.71 & -0.814 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0.884 \\ -4.17 \end{bmatrix}$$

SUMMARY

1. The system will stable when we use a suitable value of a state feed back.
2. The output response system still have a small over shot.
3. Because of measuring parameter case so it's slightly difference between simulation and experiment.
4. By adding one more direction (x-y direction), the concept of this control system can be developed for control vertically position of rocket.